Answer Sheet to the Written Exam

Financial Markets

August 2011

In order to achieve the maximal grade 12 for the course, the student must excel in all three problems.

Problem 1:

This problem focuses on testing part 1 of the course's learning objectives, that the students show "The ability to readily explain and discuss key theoretical concepts and results from academic articles, as well as their interpretation." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) Draw on Section 7.2 in the textbook. When investors are risk averse, we could expect assets with greater liquidity risk to have lower prices (offering higher return). Acharya and Pedersen (2005) see illiquidity costs as a risky asset pricing factor.

(b) Confer page 87 of the textbook. Dealers offer liquidity to the market by being willing buyers and sellers, more so when the spread is lower. Dealers earn their revenue from the spread, buying at lower prices and selling at higher prices. Intuitively, imperfect competition allows dealers to earn higher profits, stemming from a wider spread.

(c) Draw on Section 8.2.1 in the textbook, in particular page 140. Suppose that a market buy order arrives to the market. It is cleared through a match with the best available limit sell order. When the best available limit sell order is taken off the book,sellers find it more attractive to use limit orders relative to market order. Thus, the first market buy order is less likely to be followed by a market sell order.

Problem 2:

This problem focuses on testing part 2 of the course's learning objectives, that the students show "The ability to carefully derive and analyze results within an advanced, mathematically specified theoretical model." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) Note that

$$1 - \Pr(V = 1|I) = \frac{(1-\theta)\Pr(I|V=0)}{\theta \Pr(I|V=1) + (1-\theta)\Pr(I|V=0)},$$

and divide the two fractions, noticing that they have identical denominators.

(b) Market makers know nothing about the manipulator. They expect to see only the usual informed and uninformed traders, and they expect every informed trader to make the correct choice (buy when $V = 1$, sell when $V = 0$). By assumption, trader types are i.i.d., conditionally on V. It follows that trade types are i.i.d., conditionally on V. The statistical independence implies that the probability of (B_1, S_2) is the product of the separate probabilities for B_1 and S_2, all conditional on V.

The different probabilities arise from the informed traders selling when $V = 0$ but not when $V = 1$. Precisely, $\Pr(S_2|V = 0) = \alpha + (1-\alpha)(1-\beta)$ and $\Pr(S_2|V = 0) = (1-\alpha)(1-\beta)$.

Now, $a_1 = \Pr(V = 1|B_1)$ and $b_2 = \Pr(V = 1|B_1, S_2)$. Using (1),

$$\begin{aligned}\frac{\Pr(V=1|B_1,S_2)}{1-\Pr(V=1|B_1,S_2)} &= \frac{\theta\Pr(B_1,S_2|V=1)}{(1-\theta)\Pr(B_1,S_2|V=0)} = \frac{\theta\Pr(B_1|V=1)}{(1-\theta)\Pr(B_1|V=0)}\frac{\Pr(S_2|V=1)}{\Pr(S_2|V=0)}\\ &< \frac{\theta\Pr(B_1|V=1)}{(1-\theta)\Pr(B_1|V=0)} = \frac{\Pr(V=1|B_1)}{1-\Pr(V=1|B_1)}.\end{aligned}$$

Since $p/(1-p)$ is increasing, $\Pr(V = 1|B_1) > \Pr(V = 1|B_1, S_2)$, as desired.

The suggested manipulation strategy would buy one unit at price a_1 and sell it at b_2. The profit is $b_2 - a_1 < 0$. Intuitively, in the less than perfectly liquid market, there is a loss to such a round-trip trade. In particular, the sale at time 2 is interpreted as bad news about the asset value by the market maker, who therefore conducts the transaction at a lower price than the time 1 transaction.

(c) The conditional independence assumptions again imply that the conditional probability of history (B_1, h, S_T) is the product of the separate conditional probabilities for B_1, h and S_T.

The i.i.d. assumption implies that trades at different points in time are identically distributed, conditionally on V. Hence, $\Pr(B_1, h, S_T|V) = \Pr(B_1, S_2, h|V)$ for either $V \in \{0, 1\}$. By (1), then

$$\begin{aligned}\frac{\Pr(V=1|B_1,h,S_T)}{1-\Pr(V=1|B_1,h,S_T)} &= \frac{\theta\Pr(B_1,h,S_T|V=1)}{(1-\theta)\Pr(B_1,h,S_T|V=0)}\\ &= \frac{\theta\Pr(B_1,S_2,h|V=1)}{(1-\theta)\Pr(B_1,S_2,h|V=0)} = \frac{\Pr(V=1|B_1,S_2,h)}{1-\Pr(V=1|B_1,S_2,h)}.\end{aligned}$$

So, $\Pr(V = 1|B_1, h, S_T) = \Pr(V = 1|B_1, S_2, h)$.

(d) In terms of information, the set $J = (B_1, S_2, h)$ includes $I = (B_1, S_2)$. Inserting this in the law of iterated expectation gives the desired equation.

As reminded early in the problem, since $V \in \{0, 1\}$, we have $E[V] = \Pr(V = 1)$. The law of iterated expectation then translates to $E[\Pr(V = 1|B_1, S_2, h)|B_1, S_2] = \Pr(V = 1|B_1, S_2)$.

The manipulator knows that the sequence of trades will be (B_1, h, S_T). He has no special knowledge about the realization of h, but shares the market makers' beliefs about how h depends on V. [In particular, conditional independence implies that the known realizations of trades at times 1 and T do not alter the distribution of the trades h.] Hence,

$$E[b_T|B_1, S_T] = E[E[V|B_1, h, S_T]|B_1, S_T] = E[\Pr(V=1|B_1, S_2, h)|B_1, S_2] = \Pr(V=1|B_1, S_2),$$

which is the same as b_2 from (b).

The expected profit from the manipulation strategy is $E[b_T|B_1, S_T] - a_1 = b_2 - a_1 < 0$, just as in (b). The reason is the same as before, in that the trades h occurring between 2 and $T-1$ do not tend to push prices in any particular direction.

(e) I apologize for a mistake in this question. If β is large, close to one, the claim may not be true. In that case, it's possible to prove that the opposite manipulation strategy Sell followed by two herd sellers and then a Buy will be profitable. But this was not the intention of the question. Grading the answers, it will be taken into account that attempts to answer this question may have taken considerable time.

The question should have assumed $\beta = 1/2$. Using (1), like in (b),

$$\begin{aligned}
\frac{\Pr(V = 1|B_1, B_2, B_3, S_4)}{1 - \Pr(V = 1|B_1, B_2, B_3, S_4)} &= \frac{\theta \Pr(B_1, B_2, B_3, S_4|V = 1)}{(1-\theta)\Pr(B_1, B_2, B_3, S_4|V = 0)} \\
&= \frac{\Pr(V = 1|B_1)}{1 - \Pr(V = 1|B_1)} \left(\frac{\Pr(B_2|V = 1)}{\Pr(B_2|V = 0)}\right)^2 \frac{\Pr(S_4|V = 1)}{\Pr(S_4|V = 0)}.
\end{aligned}$$

The goal is to verify that

$$1 < \left(\frac{\Pr(B_2|V = 1)}{\Pr(B_2|V = 0)}\right)^2 \frac{\Pr(S_4|V = 1)}{\Pr(S_4|V = 0)} = \left(\frac{\alpha + (1-\alpha)\beta}{(1-\alpha)\beta}\right)^2 \frac{(1-\alpha)(1-\beta)}{\alpha + (1-\alpha)(1-\beta)}.$$

When $\beta = 1/2$, the latter expression is

$$\left(\frac{\alpha + (1-\alpha)/2}{(1-\alpha)/2}\right)^2 \frac{(1-\alpha)/2}{\alpha + (1-\alpha)/2} = \frac{\alpha + (1-\alpha)/2}{(1-\alpha)/2} > 1$$

as desired.

Problem 3:

This problem focuses on testing part 3 of the course's learning objectives, that the students show "The ability to apply the most relevant theoretical apparatus to analyze a given, new case-based problem." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

Below are some suggested applications of the course literature to this case. It is important to note that these applications have shortcomings which should be discussed.

- The main idea of listing a company's stock at an exchange is to facilitate trading among investors. Liquidity arises where traders stand ready to be counterparties, as happens at exchanges. If it's true that an exchange with great market power would increase the company's cost of listing, then we can presume that more companies stay off the exchange, and have have less liquid stocks.
- Chapters 6 and 9 note that illiquidity tends to have the effect that prices vary more around the efficient price.
- Chapter 1 and Section 10.3 discuss the effects of exchange competition on liquidity. It could be particularly relevant to know whether exchange consolidation would make the market making business more or less competitive. The text does not discussion this problem.
- Section 10.1 discusses some effects of trade transparency. The text implicitly presumes that competition among exchanges will provide more proper forms of transparency. Despite the relatively mixed theoretical results of Section 10.1, the "proper" provision of information may imply that it's good for liquidity. It is not obvious, however, whether competition really drives exchanges in this direction.